One Sample

Sample Mean vs. Population Mean

Two Tails:

$$H_0 : \mu = cons$$
$$H_A : \mu \neq cons$$

One Tail:

$$H_0 : \mu \leq cons$$
$$H_A : \mu > cons$$

$where : \bar{x} : sample\ \ mean,\ \mu : pop\ \ mean$

Case 1: Z test

1. The data are normally distributed, and the pop standard deviation (σ) is known.
2. The data are not normally distributed, but sample size (n) is large and σ is known; the test stat is:

$$z = \frac{\bar{x} - cons}{\sigma / \sqrt{n}}$$

3. The data are not normally distributed, n is large, and σ is unknown. We use sample standard deviation s instead of σ. The test stat is:

$$z = \frac{\bar{x} - cons}{s / \sqrt{n}}$$

If two tails: p-value = 2(1-NORMDIST(Z))
If one tails: p-value = 1-NORMDIST(Z)

Case 2: t test

4. n is small (usually less than 120) and σ is unknown; the test stat with (n-1) degrees of freedom

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}}$$

If two tails: p-value = TDIST(t, n-1,2)
If one tails: p-value = TDIST(t, n-1,1)

Sample Proportion vs. Population Proportion

Two Tails:

$$H_0 : \pi = cons$$
$$H_A : \pi \neq cons$$

One Tail:

$$H_0 : \pi \leq cons$$
$$H_A : \pi > cons$$

$where : p : sample\ \ prop,\ \pi : pop\ \ prop$

Using Z test only

(Note: this is the rule for all hypothesis tests of proportion)

$$z = \frac{p - cons}{\sqrt{\frac{cons(1 - cons)}{n}}}$$

If two tails:
p-value = 2(1-NORMDIST(Z))
If one tails:
p-value = (1-NORMDIST(Z))

Two Sample

Difference between Two Sample Means

Two Tails:

$$H_0 : \bar{\mu}_1 = \bar{\mu}_2$$
$$H_A : \bar{\mu}_1 \neq \bar{\mu}_2$$

One Tail:

$$H_0 : \bar{\mu}_1 \leq \bar{\mu}_2$$
$$H_A : \bar{\mu}_1 > \bar{\mu}_2$$

Case 1: Z test

1. σ_1 and σ_2 are known:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

2. σ_1 and σ_2 are unknown:

2a. sample size (n) is large;

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Case 2: t test

2b. n is small and $\sigma_1 \neq \sigma_2$,

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

2c. n is small and $\sigma_1 = \sigma_2$, we can use the above formula or use **pooled variance** s_p^2:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Difference between Two Sample Proportions

Two Tails:

$$H_0 : \pi_1 = \pi_2$$
$$H_A : \pi_1 \neq \pi_2$$

One Tail:

$$H_0 : \pi_1 \leq \pi_2$$
$$H_A : \pi_1 > \pi_2$$

Using Z test only

$$z = \frac{p_1 - p_2}{\sqrt{\left(\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)}}$$

(Note: in $\sqrt{\ }$, pop proportion, not sample proportion)

Conclusion: if p value is less than 0.05 *(this is the typical critical value we often choose; sometimes we could choose 0.1 or 0.01 as our critical values)*, we reject the null hypothesis (H_0);
Otherwise, we fail to reject the null hypothesis (H_0).

Source: Mari Yamamoto, PhD candidate, School of Public Affairs, Summer 2003.